

SOFT SUPERSYMMETRY-BREAKING TERMS AND THE μ PROBLEM*

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ABSTRACT

The connection between Supergravity and the low-energy world is analyzed. In particular, the soft Supersymmetry-breaking terms arising in Supergravity, the μ problem and various solutions proposed to solve it are reviewed. The soft terms arising in Supergravity theories coming from Superstring theory are also computed and the solutions proposed to solve the μ problem, which are naturally present in Superstrings, are also discussed. The \bar{B} soft terms associated are given for the different solutions. Finally, the low-energy Supersymmetric-spectra, which are very characteristic, are obtained.

FTUAM 95/20
July 1995

*Based on talks given at the Warsaw-Boston Workshop on "Physics from Planck scale to Electroweak scale", Warsaw, September 1994; Korea Physical Society Fall Meeting, Kyung-ju, October 1994; International Conference "Beyond the Standard Model IV", Lake Tahoe (California), December 1994; International Workshop on "Elementary Particle Physics Present and Future", Valencia, June 1995.

[†]Research supported in part by: the CICYT, under contract AEN93-0673; the European Union, under contracts CHRX-CT93-0132 and SC1-CT92-0792.

1. Introduction and Summary

The particle spectrum in Supersymmetric versions of the Standard Model (SM) is in general determined by soft Supersymmetry (SUSY)-breaking mass parameters like gaugino, squark, and slepton masses. The possible numerical values of these soft terms are only constrained by experimental bounds on SUSY-masses or some indirect theoretical arguments. If low-energy Supersymmetry is correct, eventually (and hopefully) the spectrum of SUSY-particles will be measured and the structure of soft SUSY-breaking terms will be subject to experimental test. Since the breaking of Supergravity (SUGRA) generates the soft terms one could, in principle, calculate explicitly their values. However, the functions which determine the SUGRA theory are somehow arbitrary and therefore the soft terms become SUGRA model-dependent. On the other hand, SUGRA theories coming from Superstring theory are much more constrained and this will provide a theory of soft terms which could enable us to interpret the (future) experimental results on SUSY-spectra. The purpose of this section is to review this approach for addressing the problem. After reviewing the theoretical considerations which lead to the introduction of SUSY, the soft SUSY-breaking terms are briefly analyzed in connection with SUGRA. Finally, after discussing the relevant theoretical arguments that can be given in favour of Superstring theory, the connection of this with SUGRA and therefore with the soft terms is expounded.

1.1. *Why Supersymmetry?*

Despite the absence of experimental verification, relevant theoretical arguments can be given in favour of SUSY^{1,2}:

- i) It is a new symmetry which relates bosons and fermions.

The importance of this is twofold: we know from the past that symmetries are crucial in particle physics and, besides, SUSY implies a new kind of unification, between particles of different spin. The latter involves that the Higgs is no longer a mysterious particle as it stands in the SM, the only fundamental scalar particle which should exist. Now, the Supersymmetric Standard Model (SSM) is naturally full of fundamental scalars (squarks, sleptons and Higgses) related through SUSY with their fermionic partners (quarks, leptons and Higgsinos).

- ii) The local version of SUSY leads to a partial unification of the SM with gravity, SUGRA^{3,2,4}.

- iii) String theory^{5,6,7} needs to be SUSY (Superstring) in order to avoid tachyons in the spectrum and, besides, after compactification of extra dimensions leads to an effective SUGRA.

- iv) Whereas the quadratic and quartic terms of the Higgs potential which are necessary in order to break the electroweak symmetry have to be postulated "ad hoc" in the case of the SM, they appear in a natural way in the context of the SSM.

- v) The joining of the three gauge coupling constants of the SM at a single uni-

fication scale which only taking into account SUSY agrees with the experimental results.

But still, the most important argument in favour of SUSY, is that

vi) It solves the so-called gauge hierarchy problem.

If we believe that the SM should be embedded within a more fundamental theory including gravity with a characteristic scale of order the Planck mass, $M_P \equiv G_N^{-1/2}/\sqrt{8\pi} \simeq 2.4 \times 10^{18} GeV$, then we are faced with the hierarchy problem. There is no symmetry protecting the masses of the scalar particles against quadratic divergences in perturbation theory. Therefore they will be proportional to the huge cut-off scale $\sim M_P$. The Higgs particle is included in the SM because of its good properties: can have vacuum expectation value (VEV), without breaking Lorentz invariance, inducing the spontaneous breaking of the electroweak symmetry at the same time that generates the fermion masses through Yukawa couplings. But, of course, all these properties are due to the fact that the Higgs is a scalar particle. As mentioned above, this leads to a huge mass for it and as a consequence for the W and Z gauge bosons. This problem of naturalness, to stabilize $M_W \ll M_P$ against quantum corrections, is solved in SUSY since now the scalar mass and the mass of its superpartner, the fermion, are related. As a consequence, only a logarithmic divergence in the scalar mass is left. In diagrammatic language, the dangerous diagrams of SM particles are cancelled with new ones which are present due to the existence of the additional partners and couplings.

1.2. Why soft Supersymmetry-breaking terms?

SUSY cannot be an exact symmetry of nature and should be broken since a SUSY particle with the same mass than its SM partner has never been detected (e.g. there is no selectron with mass $\sim 0.5 MeV$). One may introduce terms in the Lagrangian which explicitly break SUSY but, they should not induce quadratic divergences in order not to spoil the SUSY solution to the gauge hierarchy problem. This restricted type of terms exists, they have been completely listed⁸, and they are called soft SUSY-breaking terms. The simplest SSM is the so-called Minimal Supersymmetric Standard Model (MSSM), where the matter consists of three generations of quark and lepton superfields plus two Higgs doublets (SUSY demands the presence of two Higgs doublets unlike the SM where only one is needed), H_1 and H_2 of opposite hypercharge, and the gauge sector consists of $SU(3) \times SU(2)_L \times U(1)_Y$ vector superfields. Assuming certain universality of soft terms, these can be parametrized by only four parameters: a universal gaugino mass (where the gauginos are the superpartners of the gauge bosons) M , a universal scalar mass m , a universal trilinear scalar parameter (associated with the Yukawa couplings) A and an extra bilinear scalar parameter (associated with possible bilinear couplings) B .

The soft terms are very important since they determine the SUSY-spectrum, like gaugino, squark and slepton masses, and contribute to the Higgs potential generating the radiative breakdown of the electroweak symmetry. Although in principle they are

constrained because they must reproduce the experimental results ($M_W \simeq 80\text{GeV}$), the correct numbers can be obtained for wide ranges of the above parameters. In this sense the predictivity of the theory is limited.

1.3. Supergravity as the origin of the soft terms

The previous mechanism for breaking SUSY explicitly looks arbitrary but it turns out to be the most natural one when global SUSY is promoted to local, i.e. in SUGRA. When SUGRA is spontaneously broken in a "hidden" sector the soft SUSY-breaking terms are generated (see e.g. refs.^{9,10} and references therein). The process is the following: singlet scalar fields under the observable gauge group ("hidden" sector fields) acquire VEVs giving rise to spontaneous breaking of SUGRA. The goldstino, which is a combination of the fermionic partners of the above fields, is swallowed by the gravitino (the spin 3/2 superpartner of the graviton in $N = 1$ SUGRA) which becomes massive. This is the so-called super-Higgs effect. It is completely analogous to the usual Higgs mechanism. Then, the hidden fields, which only have gravitational interactions with the observable sector, decouple from the low-energy theory and the only signals they produce are the soft terms. These are characterized by the gravitino mass ($m_{3/2}$) scale and therefore in order not to introduce a new problem of naturalness, $m_{3/2}$ should be of the electroweak scale order (recall that the soft terms contribute to the Higgs masses). An interesting non-perturbative source of SUSY-breaking, capable of generating this large mass hierarchy ($m_{3/2} \ll M_P$), is gaugino condensation in some hidden sector gauge group^{11,12}. Below M_P (i.e. in the so-called flat limit where $M_P \rightarrow \infty$ but $m_{3/2}$ is kept fixed), one is left with a global supersymmetric Lagrangian plus the soft SUSY-breaking terms. In summary, the MSSM is just an effective low-energy theory derived from SUGRA when this is spontaneously broken.

Two main arguments can be used in order to criticize the previous SUSY-breaking mechanism: First of all, SUGRA is a non-renormalizable theory and then it is not clear that we are doing a consistent analysis. Notice however that the effective Lagrangian below the Planck scale is renormalizable and we are interested only in this region. The general idea below this approach is that we are considering the SUGRA Lagrangian as an effective phenomenological Lagrangian which comes from a bigger structure, renormalizable or even finite (Superstring theory?). The situation is similar to the one of the old Fermi Theory. Second, the existence of the hidden sector has to be postulated "ad hoc". However, we will see below that in the context of Superstring theory it appears in a natural way.

The full SUGRA Lagrangian¹³ is specified in terms of two functions which depend on the hidden and observable scalars of the theory: the real gauge-invariant Kähler function G which is a combination of two functions K and W , and the analytic gauge kinetic function f . K is the Kähler potential whose second derivative determines the kinetic terms for the fields in the chiral supermultiplets and is thus important for obtaining the proper normalization of the fields. W is the complete analytic super-

potential which is related with the Yukawa couplings (which eventually determine the fermion masses) and also includes possibly non-perturbative effects. Finally, f determines the kinetic terms for the fields in the vector supermultiplets, and in particular the gauge coupling constant $Ref_a = 1/g_a^2$. The subindex a is associated with the different gauge groups of the theory $G = \prod_a G_a$. For example, in the case of the SM coupled to SUGRA it is associated with $SU(3)$, $SU(2)_L$, $U(1)_Y$. Then, once we know these functions the soft SUSY-breaking terms are calculable. Unfortunately for the predictivity of the theory, G and f are arbitrary and therefore *the soft terms become SUGRA model-dependent*. All the above mentioned problems can be solved in Superstring theory.

1.4. Why Superstring theory?

As in the case of SUSY, relevant theoretical arguments can be given in favour of Superstring theory:

- i) It is possible to obtain models resembling the SSM at low-energy^{14,15}.

Of course, this is crucial in order to connect Superstring theory with the observable world.

- ii) It is the only hope to answer fundamental questions that in the context of the SM, SSM or Grand Unified Theories (GUTs) cannot even be posed: why the gauge and Yukawa couplings should have a particular value?.

First, the gauge coupling constants are dynamical because they arise as the VEV of a gauge singlet field S called the dilaton, $\langle ReS \rangle = 1/g_a^2$. This result can be understood taking into account that in Superstring theory $f_a \simeq S$ (at tree level¹⁶). It is worth noticing here that the gauge coupling constants are unified[†] even in the absence of a GUT. Thus GUT gauge groups, as e.g. $SU(5)$ or $SO(10)$, are not mandatory in order to have unification in the context of Superstring theory. Second, the Yukawa couplings, which determine the quark and lepton masses, can be explicitly calculated^{19,20} and they turn out to be also dynamical. They depend in general on other gauge singlet fields T_m called the moduli whose VEVs determine the size and shape of the compactified space. E.g. for the overall modulus $\langle ReT \rangle \sim R^2$. In fact, particular values of these fields allow us to reproduce, in principle, the peculiar observed pattern of quark and lepton masses and mixing angles²⁰. Since general experimental data (values of the gauge couplings and no observation of extra dimensions) demand $\langle ReS \rangle \sim 2$ and $\langle ReT \rangle \sim 1$ (in M_P units), the initial questions translate as how are the VEVs determined. This will be discussed below.

[†]The natural unification scale in Superstring models is¹⁷ $M_{string} \simeq 0.5 \times g_{string} \times 10^{18}$ GeV, where $g_{string} = (ReS)^{-1/2} \simeq 0.7$. However, the (apparent success of the) joining of gauge coupling constants at high energies, with the particle content of the MSSM, takes place at a scale $M_X \simeq 3 \times 10^{16}$ GeV. Although this unification takes place at energies around a factor $\simeq 12$ smaller than expected in Superstring theory, several mechanisms have been proposed in order to explain this discrepancy¹⁸.

Finally, the most outstanding virtue of Superstring theory is that

iii) It is the only (finite) theory which can unify all the known interactions including gravity.

1.5. Superstring theory as the origin of Supergravity

The ten-dimensional Heterotic Superstring²¹ after compactification of six extra dimensions (on some compact manifolds) leads to a $N = 1$ effective SUGRA. Now, f and K are in principle calculable^{16,22,17,23,24,25} from Superstring scattering amplitudes. Besides, whereas in SUGRA (non-Superstring) models we do not have the slightest idea of what fields could be involved in SUSY-breaking, four-dimensional *Superstring theory automatically has natural candidates for that job*: the dilaton S and the moduli T_m . These gauge singlet fields are generically present in four-dimensional Heterotic Superstrings since S is related with the gravitational sector of the theory and T_m are related with the extra dimensions. While other extra fields could also play a role in specific models, the dilaton and moduli constitute in some way the *minimal possible SUSY-breaking sector in Superstring theory*[§].

Concerning the superpotential W , the situation is more involved. It is known that the process of SUSY-breaking in Superstring theory has to have a non-perturbative origin since SUSY is preserved order by order in perturbation theory and hence S and T_m are undetermined at this level (the scalar potential $V(S, T_m)$ is flat). On the other hand, very little is known about non-perturbative effects in Superstring theory, particularly in the four-dimensional case. It is true that gaugino condensation gives rise to an effective $W(S, T_m)$ which breaks SUSY at the same time that S and T_m acquire reasonable VEVs, as the ones explained in point (ii) (see e.g. ref.²⁷ and references therein), and determines explicitly the values of the soft SUSY-breaking terms²⁸. However one should keep in mind two caveats. First, the VEV of the scalar potential, i.e. the cosmological constant (for a review of the cosmological constant problem, see ref.²⁹), is non-vanishing (negative). For an extended discussion on this point see section 8 of ref.²⁶. Second, this analysis requires the assumption that the dominant non-perturbative effects in Superstring theory are the field theory ones. This is because gaugino condensation is not a pure "stringy" mechanism. Thus a pessimist would say that Superstring theory does not look particularly promising in trying to get information about the SUSY-breaking sector of the theory.

However, since K and f are known in Superstring models, and the degrees of freedom involved in the process of SUSY-breaking have been identified (the hidden fields S and T_m), the effect of SUSY-breaking can be parametrized by the VEVs of the auxiliary fields of the degrees of freedom identified without specifying what is the origin of SUSY-breaking. This situation is similar to the one concerning $SU(2)_L \times U(1)_Y$ breaking in the SM. We do not really know for sure how the gauge symmetry of the SM is broken. We just parametrize our ignorance by using a Higgs field (either

[§]Starting with this minimal sector one can also study the possible role on SUSY-breaking of other extra fields (see the example discussed in section 8 of ref.²⁶).

composite or elementary) with a non-vanishing VEV, the key ingredient here is knowing the degrees of freedom (an $SU(2)_L$ doublet) involved in the process of symmetry breaking. Once we know that, we can obtain all the experimentally confirmed predictions of the SM. The purpose of the present paper is to review the above approach for addressing the problem, trying to provide a theory of soft terms which could enable us to interpret the (future) experimental results on SUSY-spectra.

The structure of the paper is as follows. In sect.2 we discuss some general formulae for the computation of soft terms in SUGRA. We analyze how the existence of hidden sector dependent masses and couplings modifies the usual soft terms taking a simple case as a guiding example. Finally, we also discuss the so-called μ problem and various solutions proposed to generate a μ term in the low-energy theory. The B soft term is given for the different solutions. In sect.3 we apply the formulae obtained in the previous section to the computation of soft terms in Superstring theory^{30,31,32,27,28,33,26,34}. It turns out to be specially useful to introduce a "goldstino angle" whose value tells us where the dominant source of SUSY-breaking resides²⁶. All formulae for soft parameters take on particularly simple forms when written in terms of this variable. We also allow for a non-vanishing vacuum energy and arbitrary complex phases in the relevant VEVs. Allowing for these turns out to be important for some relevant issues concerning soft terms. We also discuss the more model-dependent B soft term for the solutions to the μ problem which were analyzed in the previous section. This kind of solutions are naturally present in Superstring theory. Finally, we compute the soft terms for some large classes of models including the large-radius limit of Calabi-Yau-type-compactifications³⁵ and orbifold-type models³⁶. The low-energy renormalization group running of soft terms, the low-energy sparticle-spectra and the appropriate radiative $SU(2)_L \times U(1)_Y$ breaking are considered. This analysis leads to specific patterns for the SUSY-spectra which could be tested in future colliders.

2. Soft terms from Supergravity and the μ problem

2.1. General structure of soft terms

Soft scalar masses, trilinear and bilinear scalar terms arise from the expansion of the SUGRA scalar potential

$$V = e^G [G_\alpha (G^{-1})^\alpha_\beta G^\beta - 3] \quad (1)$$

and soft gaugino masses for the canonically normalized gaugino fields can be obtained from the fermionic part of the SUGRA Lagrangian¹³

$$M_a = \frac{1}{2} (Re f_a)^{-1} e^{G/2} f_a^\alpha (G^{-1})^\beta_\alpha G_\beta , \quad (2)$$

where the real gauge-invariant Kähler function G is given by

$$G(z_\alpha, z_\alpha^*) = K(z_\alpha, z_\alpha^*) + \log |W(z_\alpha)|^2 \quad (3)$$

and we use from now on the standard SUGRA mass units where $M_P = 1$ and the standard SUGRA conventions on derivatives (e.g. $G_\alpha = \frac{\partial G}{\partial z_\alpha^*}$, $G^\alpha = \frac{\partial G}{\partial z_\alpha}$). K and W are given in general by the form

$$K = K_0(h_l, h_l^*) + K_{ij}\phi_i\phi_j^* + (Z_{ij}\phi_i\phi_j + h.c.) + \dots, \quad (4)$$

$$W = W_0(h_l) + \mu_{ij}\phi_i\phi_j + Y_{ijk}\phi_i\phi_j\phi_k + \dots, \quad (5)$$

where we assume two different types of scalar fields $z_\alpha = h_l, \phi_i$. ϕ_i correspond to the observable sector (they include the SSM fields) and h_l correspond to a hidden sector. The latter are responsible for the SUSY breaking when some of them acquire large ($\gg M_W$) VEVs. The ellipsis indicates terms of higher order in ϕ_i, ϕ_i^* and the quantities $\mu_{ij}, Y_{ijk}, K_{ij}$ and Z_{ij} are in general h_l, h_l^* dependent.

Then, the form of the effective soft Lagrangian obtained from eqs.(1,2) is given in general by

$$\mathcal{L}_{soft} = \frac{1}{2} \sum_a M_a \bar{\lambda}_a \hat{\lambda}_a - \sum_i m_i^2 |\hat{\phi}_i|^2 - (A_{ijk} \hat{Y}_{ijk} \hat{\phi}_i \hat{\phi}_j \hat{\phi}_k + B_{ij} \hat{\mu}_{ij} \hat{\phi}_i \hat{\phi}_j + h.c.) . \quad (6)$$

The mass parameter $\hat{\mu}$ is related with Z and μ terms of eqs.(4) and (5) respectively and will be discussed below in the context of the so-called μ problem. We recall that the passage to the effective low-energy theory involves a number of rescalings. In particular, in the previous equation

$$\begin{aligned} \hat{\phi}_i &= K_i^{1/2} \phi_i, \\ \hat{\lambda}_a &= (Ref_a)^{1/2} \lambda_a, \\ \hat{Y}_{ijk} &= Y_{ijk} \frac{W_0^*}{|W_0|} e^{K_0/2} (K_i K_j K_k)^{-1/2}, \end{aligned} \quad (7)$$

where $\hat{\phi}_i, \hat{\lambda}_a$ are the scalar and gaugino canonically normalized fields respectively.[¶]

In the case of the MSSM the Kähler potential (to first order in the observable fields ϕ_i) and superpotential have the form (see eqs.(4,5))

$$K = K_0(h_l, h_l^*) + \sum_i K_i \phi_i \phi_i^* + (Z H_1 H_2 + h.c.), \quad (8)$$

$$W = W_0(h_l) + \mu H_1 H_2 + \sum_{generations} (Y_u Q_L H_2 u_L^c + Y_d Q_L H_1 d_L^c + Y_e L_L H_1 e_L^c), \quad (9)$$

where now $i = Q_L, u_L^c, d_L^c, L_L, e_L^c, H_1, H_2$. These equations include the usual Yukawa couplings ($Y_{ijk} = Y_u, Y_d, Y_e$, in a self-explanatory notation) and we have allowed a possible mass μ and coupling Z for the Higgses (recall that they have opposite hypercharges), where μ and Z are in principle free parameters. Finally, the subindex a in

[¶]For phenomenological reasons related to the absence of flavour changing neutral currents (FCNC) in the effective low-energy theory (see section 5 of ref.²⁶ for a discussion on this point) from now on we will assume a diagonal form for the part of the Kähler potential associated with matter fields, $K_{ij} = K_i \delta_j^i$ in eq.(4).

eq.(2) is associated with the different gauge groups of the theory, i.e. $SU(3)$, $SU(2)_L$ and $U(1)_Y$.

As already explained in the Introduction, the particular values of the soft terms depend on the type of SUGRA theory from which the MSSM derives and, in general, on the mechanism of SUSY-breaking. But, in fact, is still possible to learn things about soft terms without knowing the details of SUSY-breaking. Let us consider the simple case of canonical kinetic terms for hidden and observable fields^{37,9} (i.e. $K_0 = \sum_l h_l h_l^*$ and $K_i = 1$ in eq.(8)) and $Z = 0$. Then, irrespective of the SUSY-breaking mechanism, the scalar masses and the A , B terms can be straightforwardly computed:

$$m_i^2 = m_{3/2}^2 + V_0 , \quad (10)$$

$$A_{ijk} = m_{3/2} \sum_l G_{0h_l} \left(h_l^* + \frac{Y_{ijk}^{h_l}}{Y_{ijk}} \right) , \quad (11)$$

$$B_\mu = m_{3/2} \sum_l G_{0h_l} \left(h_l^* + \frac{\mu^{h_l}}{\mu} \right) - m_{3/2} , \quad (12)$$

where $m_{3/2}^2 = e^{G_0}$ is the gravitino mass and $V_0 = e^{G_0}(\sum_l |G_{0h_l}|^2 - 3)$ is the VEV of the scalar potential (i.e. the cosmological constant). The latter has a bearing on measurable quantities like scalar masses and therefore the way we deal with the cosmological constant problem is important. Anyway, the scalar masses are automatically universal in this case. This also happens for the A terms assuming that the Yukawa couplings, Y_{ijk} , are hidden field independent. Besides, the gaugino masses are universal if the gauge kinetic function is the same for the different gauge groups of the theory $f_a = f$ (note that e.g. the case of a constant f is not phenomenologically interesting since it would imply $M_a = 0$ as can be seen from eq.(2)). Finally, a particularly interesting value of B can be obtained assuming also that μ is h_l independent and the cosmological constant is vanishing. Then, from eqs.(11,12) the well known result for the B term is recovered:

$$B_\mu = A - m_{3/2} , \quad (13)$$

where $A = A_{ijk}$ and we call B_μ the B term since it is associated only (recall that we are analyzing the case $Z = 0$) with the μ term of eq.(9).

This SUGRA theory is attractive for its simplicity and for the natural explanation that it offers to the universality of the soft scalar masses. Actually, universality is a desirable property not only to reduce the number of independent parameters in the MSSM, but also for phenomenological reasons, particularly to avoid flavour-changing neutral currents (FCNC) (see e.g. ref.³⁸).

However, One can think of many possible SUGRA models (with different K and f) leading to SUSY-breaking in a hidden sector of the theory *leading in turn to different results for the soft terms*. We will see in sect.3 how this problem can be ameliorated in the context of Superstring theory, where K , f and the hidden sector are more constrained.

2.2. The μ problem

Let us now discuss the μ problem and the different solutions proposed in the literature illustrating them in the case of canonical kinetic terms. From eqs.(6,10,12) and taking into account SUSY D and F-terms the relevant Higgs scalar potential along the neutral direction for the electroweak breaking is readily obtained

$$V(H_1, H_2) = \frac{1}{8}(g_2^2 + g'^2)(|H_2|^2 - |H_1|^2)^2 + m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1 H_2 + h.c.) \quad (14)$$

with

$$\begin{aligned} m_{1,2}^2 &= m_{3/2}^2 + V_0 + |\hat{\mu}|^2, \\ m_3^2 &= -B_\mu \hat{\mu}, \\ \hat{\mu} &= \mu e^{K_0/2} \frac{W_0^*}{|W_0|}, \end{aligned} \quad (15)$$

where $g_3 = g_2 = g_1 = \sqrt{5/3} g'$ at the unification scale and the Higgsino mass $\hat{\mu}$ gives a SUSY contribution (through the F-terms) to the Higgs scalar masses. This is the SUSY version of the usual Higgs potential in the SM. As mentioned in the Introduction, it appears in a natural way whereas in the SM has to be postulated "ad hoc". It must develop a minimum at $\langle H_{1,2} \rangle = \nu_{1,2}$ in such a way that $\nu_1^2 + \nu_2^2 = 2M_W^2/g_2^2$. This is the realistic minimum that corresponds to the standard vacuum.

For this scheme to work, the presence of the second term in eq.(9) is crucial. If $\mu = 0$, then the form of the renormalization group equations (RGEs) implies that such a term is not generated at any Q scale since $\mu(Q) \propto \mu$. The same occurs for m_3 , i.e. $m_3(Q) \propto \mu$. Then, the minimum of the potential of eq.(14) occurs for $\nu_1 = 0$ and, therefore, d -type quarks and e -type leptons remain massless. Besides, the superpotential of eq.(9) with $\mu = 0$ possesses a spontaneously broken Peccei-Quinn symmetry³⁹ leading to the appearance of an unacceptable Weinberg-Wilczek axion⁴⁰.

Once it is accepted that the presence of the μ -term in the superpotential is essential, there arises an immediate question: Is there any dynamical reason why μ should be small, of the order of the electroweak scale?. Note that, to this respect, the μ -term is different from the soft SUSY-breaking terms, which are characterized by the small scale $m_{3/2}$ once we assume correct SUSY breaking. In principle the natural scale of μ would be M_P , but this would re-introduce the problem of naturalness since the Higgs scalars get a contribution μ^2 to their squared mass (see eq.(15)). Thus, any complete explanation of the electroweak breaking scale must justify the origin of μ . This is the so-called μ problem. This problem has been considered by several authors and different possible solutions have been proposed^{9,41,42,43,44,45}.

(a) In ref.⁴³ was pointed out that the presence of a non-renormalizable term in the superpotential

$$\lambda W_0 H_1 H_2 \quad (16)$$

characterized by the coupling λ , which mixes the observable sector with the hidden sector, yields dynamically a μ parameter when W_0 acquires a VEV

$$\mu = \lambda W_0 . \quad (17)$$

The fact that μ is of the electroweak scale order is a consequence of our assumption of a correct SUSY-breaking scale $m_{3/2} = e^{K_0/2}|W_0| = O(M_W)$. Now, with this solution to the μ problem, eq.(12) gives (let us call B_λ the B -term associated with eqs.(16,17))

$$B_\lambda = m_{3/2} \left(2 + \frac{V_0}{m_{3/2}^2} + \sum_l G_{0h_l} \frac{\lambda^{h_l}}{\lambda} \right) \quad (18)$$

and $\hat{\mu} = m_{3/2}\lambda$ in eq.(15). For λ independent of h_l a very simple result is obtained

$$B_\lambda = m_{3/2} \left(2 + \frac{V_0}{m_{3/2}^2} \right) . \quad (19)$$

The value of A is still given by eq.(11), but the relation $B_\lambda = A - m$ (see eq.(13)) is no longer true even in the case of Y_{ijk} independent of h_l . For this mechanism to work the $\mu H_1 H_2$ term in eq.(9) must be absent (otherwise the natural scale for μ would be the Planck mass), a fact that remarkably enough, is automatically guaranteed in the framework of Superstring theory as we will see below.

(b) In refs.^{42,43} was shown that if the coupling Z is present in the Kähler potential (see eq.(8)), an effective low-energy B -term is naturally generated of the order of magnitude of the gravitino mass. We will call it B_Z . We can compute its form in our guiding example with canonical kinetic terms. The result is

$$\begin{aligned} B_Z &= \frac{m_{3/2}}{X} \left(2 + \frac{V_0}{m_{3/2}^2} + \sum_l [G_{0h_l} \frac{Z^{h_l}}{Z} - G_0^{h_l} \frac{Z_{h_l}}{Z} - |G_0^{h_l}|^2 \frac{Z_{h_l}^{h_l}}{Z}] \right) , \\ X &\equiv 1 - \sum_l G_0^{h_l} \frac{Z_{h_l}}{Z} \end{aligned} \quad (20)$$

and now the Higgsino mass $\hat{\mu}$ in eq.(15) is modified to $\hat{\mu} = m_{3/2} X Z$. Notice that the dependence on Z and λ in eqs.(20,18) is the same for the case of an analytic Z , i.e. Z independent of h_l^* . This is not a surprising result since both mechanisms for solving the μ problem are equivalent in this particular case⁴³. Indeed, in the case now considered, the SUGRA theory is equivalent to the one with a Kähler potential K (without the terms $Z H_1 H_2 + h.c.$ of eq.(8)) and a superpotential $W e^{Z H_1 H_2}$, since the function $G = K + \log |W|^2$ is the same for both. After expanding the exponential, the superpotential will have a contribution $Z W_0 H_1 H_2$, i.e. a term of the type of eq.(16). Of course, in the simple case Z also independent of h_l , B_Z coincides with the result of eq.(19)

$$B_Z = m_{3/2} \left(2 + \frac{V_0}{m_{3/2}^2} \right) . \quad (21)$$

Besides, assuming vanishing cosmological constant, i.e. $V_0 = 0$, the value of B in both cases (a), (b) is given by

$$B = 2m_{3/2} . \quad (22)$$

Notice that it is conceivable that both mechanisms could be present simultaneously. In that case the general expressions for the B -term and Higgsino mass are easily obtained

$$\begin{aligned} B &= \frac{1}{\hat{\mu}}(B_\lambda m_{3/2} \lambda + B_Z m_{3/2} X Z) , \\ \hat{\mu} &= m_{3/2} \lambda + m_{3/2} X Z , \end{aligned} \quad (23)$$

where B_λ and B_Z , X are given in eqs.(18) and (20) respectively.

(c) In refs.^{9,44} the observation was made that in the framework of any SUSY-GUT, starting again with $\mu = 0$, an effective μ term is generated by the integration of the heavy degrees of freedom. The prediction for B is once more given by eq.(22).

The solutions discussed here in order to solve the μ problem *are naturally present in Superstring theory*.^{||} In ref.⁴³ was first remarked that the $\mu H_1 H_2$ term (see eq.(9)) is naturally absent from W (otherwise the natural scale for μ would be M_P) since in SUGRA theories coming from Superstring theory mass terms for light fields are forbidden in the superpotential. Then a realistic example where non-perturbative SUSY-breaking mechanisms like gaugino-squark condensation induce superpotentials of the type (a) (see eq.(16)) was given^{41,43}. In ref.⁴⁶ the same kind of superpotential was obtained using pure gaugino condensation. It was used the fact that in some classes of four-dimensional Superstrings (orbifolds) a possible $H_1 H_2$ dependence may appear in f at one-loop. The alternative mechanism (b) in which there is an extra term in the Kähler potential (see eq.(8)) originating a μ -term is also naturally present in some large classes of four-dimensional Superstrings^{33,47,46}.

In Superstring theory, neither the kinetic terms are in general canonical nor the couplings (Y_{ijk} , λ , Z) and the mass term (μ) are independent of hidden fields. However, as we will see in subsect.3.2, it is still possible to obtain (the phenomenologically desirable) universal soft terms in the so-called dilaton-dominated limit^{33,26}. This limit is not only interesting because of that, but also because it is quite model independent (i.e. for any compactification scheme the results for the soft terms are the same). It is also remarkable that in this limit once again the value of B for the two mechanisms (a), (b) coincides with that of eq.(22). If, alternatively, we just assume that a small ($\sim M_W$) dilaton-independent mass μ is present in the superpotential, then the result for B is now given²⁶ by eq.(13) as in the case of canonical kinetic terms.

We will study in detail this issue in the next section.

3. Soft terms from Superstring theory

^{||}Another possible solution via SUSY-breaking in Superstring perturbation theory can be found in ref.⁴⁵.

3.1. General structure of soft terms

Following the Introduction and the notation of sect.2, the hidden and observable sectors are given by $h_l = S, T_m$ and ϕ_i respectively. In order to proceed we need to make some simplifying assumptions. We will comment below what changes are to be expected if the assumptions are relaxed. Amongst the moduli T_m we will concentrate on the overall modulus T whose classical value gives the size of the manifold. Apart from simplicity, this modulus is the only one which is always necessarily present in any $(0, 2)$ (but left-right symmetric) 4-D Superstring. We believe that studying the one modulus case is enough to get a feeling of the most important physics of soft terms. Anyway, the case with several moduli will be analyzed (for orbifolds) in subsect.3.3. We will disregard for the moment any mixing between the S and T fields kinetic terms. In fact this is strictly correct in all 4-D Superstrings at tree level. However, it is known that this type of mixing may arise at one loop level in some cases. On the other hand, these are loop effects which should be small and in fact can be easily incorporated in the analysis in some simple cases (orbifolds) as shown in subsection 3.3.

Under the above conditions, the functions which appear in K and W eqs.(8,9) have the following general expressions:

$$K_0 = -\log(S + S^*) + K_0(T, T^*) \quad , \quad K_i = K_i(T, T^*) \quad , \quad Z = Z(T, T^*) \quad , \quad (24)$$

$$W_0 = W_0(S, T) \quad , \quad \mu = \mu(S, T) \quad , \quad Y_{ijk} = Y_{ijk}(T) \quad . \quad (25)$$

These confirm the above comment that in Superstring theory the kinetic terms are non-canonical and the couplings are hidden field dependent. The tree-level expression for f_a for any four-dimensional Superstring, as mentioned in the Introduction, is well known, $f_a = k_a S$, where k_a is the Kac-Moody level of the gauge factor. Normally (level one case) one takes $k_3 = k_2 = \frac{3}{5}k_1 = 1$. Since a possible T dependence may appear at one-loop⁴⁸, then in general

$$f_a(S, T) = k_a S + f_a(T) \quad , \quad (26)$$

where we assume that other possible chiral fields do not contribute to SUSY-breaking. It is important to stress that this gauge kinetic function does not get further renormalized beyond one-loop and that it is therefore an exact expression at all orders⁴⁹. Finally, the cosmological constant is (see eq.(1))

$$V_0 = G_0^S |F_0^S|^2 + G_0^T |F_0^T|^2 - 3e^{G_0} \quad . \quad (27)$$

Of course, the first two terms in the right hand side of eq.(27) represent the contributions of the S and T auxiliary fields, $F_0^S = e^{G_0/2} (G_0^S)^{-1} G_0^S$ and $F_0^T = e^{G_0/2} (G_0^T)^{-1} G_0^T$.

As we will show below, it is important to know what field, either S or T , plays the predominant role in the process of SUSY-breaking. This will have relevant consequences in determining the pattern of soft terms, and therefore the spectrum of

physical particles. That is why it is very useful to define an angle θ in the following way²⁶ (consistently with eq.(27)):

$$\begin{aligned} (G_0^S)^{1/2} F_0^S &= \sqrt{3} C m_{3/2} e^{i\alpha_S} \sin\theta , \\ (G_0^T)^{1/2} F_0^T &= \sqrt{3} C m_{3/2} e^{i\alpha_T} \cos\theta , \end{aligned} \quad (28)$$

where α_S, α_T are the phases of F_0^S and F_0^T , and the constant C is defined as follows:

$$C^2 = 1 + \frac{V_0}{3m_{3/2}^2} . \quad (29)$$

If the cosmological constant V_0 is assumed to vanish, one has $C = 1$, but we prefer for the moment to leave it undetermined. As we already mentioned below eq.(10), the way one deals with the cosmological constant problem is important.

Notice that, with the above assumptions, the goldstino field which is swallowed by the gravitino in the process of supersymmetry breaking is proportional to

$$\tilde{\eta} = \sin\theta \tilde{S} + \cos\theta \tilde{T} , \quad (30)$$

where \tilde{S} and \tilde{T} are the canonically normalized fermionic partners of the scalar fields S and T (we have reabsorbed here the phases by redefinitions of the fermions \tilde{S}, \tilde{T}). Thus the angle defined above may be appropriately termed *goldstino angle* and has a clear physical interpretation as a mixing angle.

Then it is straightforward to compute the general form of the soft terms^{26,50}

$$M_a = \sqrt{3} C m_{3/2} \left[\frac{k_a \text{Re} S}{\text{Re} f_a} e^{-i\alpha_S} \sin\theta + e^{-i\alpha_T} \cos\theta \frac{f_a^T (G_0^T)^{-1/2}}{2 \text{Re} f_a} \right] , \quad (31)$$

$$m_i^2 = 2m_{3/2}^2 (C^2 - 1) + m_{3/2}^2 C^2 [1 + N_i(T, T^*) \cos^2\theta] ,$$

$$N_i(T, T^*) = \frac{3}{K_{0T}^T} \left(\frac{K_{iT} K_i^T}{K_i^2} - \frac{K_{iT}^T}{K_i} \right) = -3 \frac{(\log K_i)_T^T}{K_{0T}^T} , \quad (32)$$

$$A_{ijk} = -\sqrt{3} C m_{3/2} [e^{-i\alpha_S} \sin\theta + e^{-i\alpha_T} \cos\theta \omega_{ijk}(T, T^*)] ,$$

$$\omega_{ijk}(T, T^*) = (K_{0T}^T)^{-1/2} \left(\sum_{p=i,j,k} \frac{K_p^T}{K_p} - K_0^T - \frac{Y_{ijk}^T}{Y_{ijk}} \right) , \quad (33)$$

$$B = \frac{1}{\tilde{\mu}} \left(B_Z m_{3/2} X Z + B_\mu \mu \frac{W_0^*}{|W_0|} e^{K_0/2} \right) (K_{H_1} K_{H_2})^{-1/2} ,$$

$$\begin{aligned} B_\mu &= m_{3/2} \left[-1 - C\sqrt{3} e^{-i\alpha_S} \sin\theta \left(1 - \frac{\mu^S}{\mu} (S + S^*) \right) \right. \\ &\quad \left. + C\sqrt{3} e^{-i\alpha_T} \cos\theta (K_{0T}^T)^{-1/2} \left(K_0^T + \frac{\mu^T}{\mu} - \frac{K_{H_1}^T}{K_{H_1}} - \frac{K_{H_2}^T}{K_{H_2}} \right) \right] , \end{aligned}$$

$$B_Z = \frac{m_{3/2}}{X} \left\{ (3C^2 - 1) + C\sqrt{3} e^{-i\alpha_T} \cos\theta (K_{0T}^T)^{-1/2} \left(\frac{Z^T}{Z} - \frac{K_{H_1}^T}{K_{H_1}} - \frac{K_{H_2}^T}{K_{H_2}} \right) \right\}$$

$$\begin{aligned}
& - C\sqrt{3}e^{i\alpha_T}\cos\theta(K_{0T}^T)^{-1/2}\frac{Z_T}{Z} \\
& + C^2\mathfrak{Z}(K_{0T}^T)^{-1}\cos^2\theta\left[\frac{Z_T}{Z}\left(\frac{K_{H_1}^T}{K_{H_1}}+\frac{K_{H_2}^T}{K_{H_2}}\right)-\frac{Z_T^T}{Z}\right]\Big\}, \\
X & \equiv 1 - C\sqrt{3}e^{i\alpha_T}\cos\theta(K_{0T}^T)^{-1/2}\frac{Z_T}{Z}, \\
\hat{\mu} & = \left[m_{3/2}XZ + \mu\frac{W_0^*}{|W_0|}e^{K_0/2}\right](K_{H_1}K_{H_2})^{-1/2}, \tag{34}
\end{aligned}$$

where Ref_a are the inverse squared gauge coupling constants at the string scale. Notice that we only include one-loop effects in the computation of the soft gaugino masses. The motivation is the following: one-loop corrections to the f_a function are known explicitly in some four-dimensional strings, and moreover higher-loop corrections are vanishing as mentioned before, whereas computing the one-loop-corrected bosonic soft terms would require knowledge of the one-loop-corrected Kähler potential, whose form is not available in the general case. However, for some orbifolds models, well motivated conjectures give the form of the one-loop-corrected Kähler potential: in particular, mixing appears between the S and T kinetic terms. We will take into account such corrections in subsect.3.3. They are normally negligible, but may be important for small $\sin\theta$, as we will see in specific cases.

The above expressions become much simpler in specific four-dimensional Superstrings and/or in the large- T limit. This is the case for instance of the formula for $N_i(T, T^*)$ which looks complicated but it becomes very simple. N_i is related to the curvature of the Kähler manifold parametrized by the above Kähler potential. For manifolds of constant curvature (like in the orbifold case) the N_i are constants, independent of T . More precisely, they correspond to the modular weights of the charged fields, which are normally negative integer numbers. In more complicated four-dimensional Superstrings like those based on Calabi-Yau manifolds, the $N_i(T, T^*)$ functions are complicated expressions in which world-sheet instanton effects play an important role. In the case of $(2, 2)$ Calabi-Yau manifolds, for the large T limit it turns out that $N_i(T, T^*) \rightarrow -1$. We will come back to the evaluation of the N_i in specific Superstring models later on. Anyway, the explicit dependence of the soft masses on the N_i of each particle may produce in general a lack of universality^{32,28}. This is relevant for the issue of FCNC. For an extended discussion on this point see section 5 of ref.²⁶ and refs.^{51,52}.

The soft terms obtained in the previous analysis are in general complex. Notice that if S and T fields acquire complex vacuum expectation values, then the phases α_S, α_T associated with their auxiliary fields can be non-vanishing and the functions $\omega_{ijk}(T, T^*)$, $f_a(T)$, etc. can be complex. The analysis of this situation in connection with the experimental limits on CP-violating effects like an electric dipole moment for the neutron (EDMN) can be found in section 4 of ref.²⁶ and ref.⁵³.

Finally, the above analysis has shown that the different soft SUSY-breaking terms have all an explicit dependence on V_0 , i.e. the cosmological constant, which is con-

tained in C . We have to face this fact and do something about it.** We cannot just simply ignore it, as it is often done, since the way we deal with the cosmological constant problem has a bearing on measurable quantities like scalar masses. For an extended discussion on this point see sections 6 and 8 of ref.²⁶ and refs.^{54,55,34}.

As already explained at the end of subsect.2.2, the superpotential eq.(16) which provides a possible solution to the μ problem can naturally be obtained in the context of Superstring theory with $\lambda = \lambda(T)$ in general^{43,46}. So with this solution to the μ problem, B_μ in eq.(34) gives⁵⁰

$$B_\lambda = m_{3/2} \left[(3C^2 - 1) + C\sqrt{3}e^{-i\alpha_T} \cos\theta (K_{0T}^T)^{-1/2} \left(\frac{\lambda^T}{\lambda} - \frac{K_{H_1}^T}{K_{H_1}} - \frac{K_{H_2}^T}{K_{H_2}} \right) \right] . \quad (35)$$

The alternative mechanism (b) in which there is an extra term in the Kähler potential (see eq.(8)) originating a μ -term is also naturally present in some large classes of four-dimensional Superstrings. Indeed, in the case of some orbifold models and the large-T limit of Calabi–Yau compactifications one expect^{33,47,46}

$$Z(T, T^*) \simeq \frac{1}{T + T^*} . \quad (36)$$

3.2. The $\sin\theta = 1$ (dilaton-dominated) limit

Before going into specific classes of Superstring models, it is worth studying the interesting limit $\sin\theta = 1$, corresponding to the case where the dilaton sector is the source of all the SUSY-breaking^{33,26} (see eq.(28)). Since the dilaton couples in a universal manner to all particles, *this limit is quite model independent*. Using eqs.(31,32,33,34,35) one finds the following simple expressions for the soft terms:

$$\begin{aligned} M_a &= \sqrt{3}Cm_{3/2} \frac{k_a \text{Re}S}{\text{Re}f_a} e^{-i\alpha_S} , \\ m_i^2 &= C^2 m_{3/2}^2 + 2m_{3/2}^2 (C^2 - 1) , \\ A_{ijk} &= -\sqrt{3}Cm_{3/2} e^{-i\alpha_S} , \\ B_\mu &= m_{3/2} \left[-1 - \sqrt{3}C e^{-i\alpha_S} \left(1 - \frac{\mu^S}{\mu} (S + S^*) \right) \right] , \\ B_Z &= B_\lambda = m_{3/2} (3C^2 - 1) , \end{aligned} \quad (37)$$

where the scalar masses and the A -terms are universal, whereas the gaugino masses may be slightly non-universal since non-negligible threshold effects might be present.

Notice that the expressions for m_i and $B_\lambda(B_Z)$, using eq.(29), coincide with the ones obtained in SUGRA models with canonical kinetic terms for the matter fields

*It is worth noticing that general properties which are independent of the value of the cosmological constant can still be found (see subsect.3.3 of ref.²⁶).

eqs.(10) and (19)(eq.(21)). Notice also that the expression for B_μ obtained in the limit $\mu^S = 0$ coincides with eq.(13).

It is obvious that this limit $\sin \theta = 1$ is quite predictive. For a vanishing cosmological constant (i.e. $C = 1$), the soft terms are in the ratio $m_i : M_a : A = 1 : \sqrt{3} : -\sqrt{3}$ up to small threshold effect corrections (and neglecting phases). This will result in definite patterns for the low-energy particle spectra^{56,26} as we will see below.

3.3. Computing soft terms in specific Superstring models

In order to obtain more concrete expressions for the soft terms one has to compute the functions $N_i(T, T^*)$, $\omega_{ijk}(T, T^*)$ and $f_a(S, T)$. In order to evaluate these functions one needs a minimum of information about the Kähler potential K , the structure of Yukawa couplings $Y_{ijk}(T)$ and the one-loop threshold corrections $f_a(T)$. This type of information is only known for some classes of four-dimensional Superstrings which deserve special attention. We will thus concentrate here on two large classes of models: the large- T limit of Calabi-Yau compactifications³⁵ and orbifold compactifications³⁶. We will describe the general pattern of soft terms in these large classes of Superstring models in turn.

3.3.1. Orbifold compactifications

In the case of orbifold four-dimensional Superstrings the Kähler potential has the general form (for small $|\phi_i|$)^{16,22,23}

$$K = -\log(S + S^*) - 3\log(T + T^*) + \sum_i (T + T^*)^{n_i} \phi_i \phi_i^* , \quad (38)$$

where the n_i are normally negative integers, sometimes called modular weights of the matter fields (see ref.³² for a classification of possible modular weights of charged fields in orbifolds). For example, in the case of Z_N orbifolds the possible modular weights of matter fields are $-1, -2, -3, -4, -5$. Fields belonging to the untwisted sector have $n_i = -1$. Fields in twisted sectors of the orbifold but without oscillators have usually modular weight $n_i = -2$ (twisted associated to unrotated planes of the underlying six-torus have $n_i = -1$) and those with oscillators have $n_i \leq -3$. It is important to remark that, unlike the case of smooth Calabi-Yau models, the above T dependence does not get corrections from world-sheet instantons and is equally valid for small and large T . In fact, general orbifold models have a symmetry ("target-space duality") which relates⁵⁷ small to large ReT . This is a discrete infinite subgroup of $SL(2, \mathbf{R})$ in which T plays the role of modulus. For the overall field T here considered, the target-space duality group will be either the modular group $SL(2, \mathbf{Z})$ or a subgroup of it (in some cases, if quantized Wilson lines are present). Under $SL(2, \mathbf{Z})$ the modulus transforms like

$$T \rightarrow \frac{aT - ib}{icT + d} ; \quad ad - bc = 1 , \quad a, b, c, d \in \mathbf{Z} , \quad (39)$$

the dilaton S field is invariant at tree-level and the matter fields transform like

$$C_i \rightarrow (icT + d)^{n_i} C_i \quad (40)$$

up to constant matrices which are not relevant for the present analysis. Eq.(40) explains why the integers n_i are called modular weights. With the above transformation properties the G function is modular invariant (if the superpotential W has modular weight -3).

The threshold correction functions $f_a(T)$ (due to the contribution of massive string states in the loops) have been computed for Z_N and $Z_N \times Z_M$ orbifolds only for the $(2, 2)$ case in refs.^{17,24,25,58,59} although they are expected to be valid for more general cases (see e.g. ref.³² for a general discussion on this point). The result has the form^{††}

$$f_a(T) = -\frac{1}{16\pi^2}(b'_a - k_a\delta_{GS}) \log \eta^4(T) , \quad (41)$$

where $\eta(T)$ is the well known Dedekind function which admits the representation

$$\eta(T) = e^{-\pi T/12} \prod_{n=1}^{\infty} (1 - e^{2\pi n T}) . \quad (42)$$

In eq.(41) k_a is the Kac-Moody level of the gauge group G_a , δ_{GS} is a group independent (but model-dependent) constant and

$$b'_a = -3C(G_a) + \sum_i T_a(\phi_i)(3 + 2n_i) = b_a + 2 \sum_i T_a(\phi_i)(1 + n_i) , \quad (43)$$

where $C(G_a)$ denotes the quadratic Casimir in the adjoint representation of G_a , $T_a(\phi_i)$ is defined by $Tr(T^\alpha T^\beta) = T_a(\phi_i)\delta^{\alpha\beta}$ ($T^\alpha =$ generators of G_a in the ϕ_i representation) and the sum runs over all the massless charged chiral fields. Notice that b_a is nothing but the $N = 1$ one-loop β -function coefficient of the G_a gauge coupling and that $b'_a = b_a$ when all matter fields have modular weights $n_i = -1$ (as for untwisted states). While b'_a may be computed in terms of the effective low-energy degrees of freedom of the theory, δ_{GS} is a model-dependent quantity (usually a negative integer in the case of the overall modulus T). Its presence is associated to the cancellation of the one-loop duality anomalies of the theory.

Using eqs.(26,31,32,33,34,35,36) and the above expressions for K and $f_a(T)$ one obtains the soft terms^{26,50}

$$M_a = \sqrt{3}Cm_{3/2} \left(\frac{k_a ReS}{Ref_a} e^{-i\alpha_S} \sin\theta + e^{-i\alpha_T} \cos\theta \frac{(b'_a - k_a\delta_{GS})(T + T^*)\hat{G}_2(T, T^*)}{32\pi^3\sqrt{3}Ref_a} \right) , \quad (44)$$

^{††}For the case when the underlying six dimensional torus lattice is not assumed to decompose into a direct sum of a four-dimensional and a two-dimensional sublattice, with the latter lying in a plane left fixed by a set of orbifold twists, see ^{60,61}.

$$m_i^2 = m_{3/2}^2 C^2 (1 + n_i \cos^2 \theta) + 2m_{3/2}^2 (C^2 - 1) , \quad (45)$$

$$A_{ijk} = -\sqrt{3} C m_{3/2} [e^{-i\alpha_S} \sin \theta + e^{-i\alpha_T} \cos \theta \omega_{ijk}(T, T^*)] ,$$

$$\omega_{ijk}(T, T^*) = \frac{1}{\sqrt{3}} \left(3 + n_i + n_j + n_k - (T + T^*) \frac{Y_{ijk}^T}{Y_{ijk}} \right) , \quad (46)$$

$$B_\mu = m_{3/2} \left[-1 - C\sqrt{3} e^{-i\alpha_S} \sin \theta \left(1 - \frac{\mu^S}{\mu} (S + S^*) \right) - C e^{-i\alpha_T} \cos \theta \left(3 + n_{H_1} + n_{H_2} - \frac{\mu^T}{\mu} (T + T^*) \right) \right] , \quad (47)$$

$$B_\lambda = m_{3/2} \left[(3C^2 - 1) - C e^{-i\alpha_T} \cos \theta \left(n_{H_1} + n_{H_2} - \frac{\lambda^T}{\lambda} (T + T^*) \right) \right] , \quad (48)$$

$$B_Z = \frac{m_{3/2}}{X} [(3C^2 - 1) - C \cos \theta (e^{-i\alpha_T} (n_{H_1} + n_{H_2} + 1) - e^{i\alpha_T}) - C^2 \cos^2 \theta (2 + n_{H_1} + n_{H_2})] ,$$

$$X \equiv 1 + C e^{i\alpha_T} \cos \theta , \quad (49)$$

where \hat{G}_2 is the non-holomorphic Eisenstein function which may be defined by $\hat{G}_2 = G_2(T) - 2\pi/(T + T^*)$. Here G_2 is the holomorphic Eisenstein form which is related to the Dedekind function by $G_2(T) = -4\pi(\partial\eta(T)/\partial T)(\eta(T))^{-1}$. In fact, using eq.(31) one gets eq.(44) with G_2 instead of \hat{G}_2 . However, one gets the complete modular invariant result in eq.(44) when one includes the one-loop contribution of massless fields (see ref.³²) and the one-loop (Superstring) correction to the Kähler potential (see ref.⁵⁰).

Notice the explicit dependence of the soft masses on the modular weights of each particle. As mentioned in subsection 3.1, this lack of universality may be relevant for the issue of FCNC.

The A -parameters depend on the modular weights of the particles appearing in the Yukawa coupling. Moreover, the last term in ω_{ijk} , eq.(46), drops for Yukawa couplings involving either untwisted fields or (for large T) twisted fields associated to the *same* fixed point. The reason is that the Yukawa couplings are constants or tend exponentially to constants, respectively¹⁹. This simplify the phenomenological analysis since the relevant couplings will be of this type. (The A term which is relevant to electroweak symmetry breaking is the one associated to the top-quark Yukawa coupling. If the fields are twisted they should be associated to the same fixed point in order to obtain the largest possible value of the coupling, otherwise it would be exponentially suppressed²⁰.)

One-loop (Superstring) corrections

As discussed in subsection 3.1 there is a slight inconsistency in using one-loop formulae for the gaugino masses whereas for the other soft terms m_i , A and B we use only the tree level result. In fact this is not that important since normally the one-loop corrections are small for those terms. Anyway, they may be evaluated knowing the one-loop (Superstring) corrections to the Kähler potential. General arguments

applicable to orbifolds allow us to write the one-loop corrected Kähler potential for orbifolds by making the replacement⁵⁸

$$S + S^* \longrightarrow Y = S + S^* - \frac{\delta_{GS}}{8\pi^2} \log(T + T^*) \quad (50)$$

in eq.(38). δ_{GS} measures the amount of one-loop mixing between the S and T fields in the Kähler potential. This one-loop mixing term with coefficient δ_{GS} generalizes the Green–Schwarz mechanism⁶² and cancels anomalies of the underlying non-linear σ -model^{58,25,63}, which are described by triangle diagrams with two external gauge bosons and several external moduli fields T . This type of mixing is expected to be present in generic four-dimensional strings and not only in the orbifold case. Now, for the one-loop Kähler potential to transform in the required way under target-space duality transformations the dilaton has to acquire a non-trivial modular transformation behaviour at the one-loop level⁵⁸

$$S \rightarrow S - \frac{\delta_{GS}}{8\pi^2} \log(icT + d) . \quad (51)$$

The expression (27) for the VEV of the scalar potential gets modified as follows²⁶:

$$\begin{aligned} V_0 &= e^{G_0} Y^2 |G_0^S|^2 + e^{G_0} \frac{(T + T^*)^2}{3} \left(1 - \frac{\delta_{GS}}{24\pi^2 Y} \right)^{-1} \left| G_0^T + \frac{\delta_{GS}}{8\pi^2 (T + T^*)} G_0^S \right|^2 - 3e^{G_0} \\ &= \frac{1}{Y^2} \left| F_0^S - \frac{\delta_{GS}}{8\pi^2 (T + T^*)} F_0^T \right|^2 + \frac{3}{(T + T^*)^2} \left(1 - \frac{\delta_{GS}}{24\pi^2 Y} \right) |F_0^T|^2 - 3e^{G_0} . \end{aligned} \quad (52)$$

Analogously to eq.(27), we have written V_0 also in terms of the S and T auxiliary fields F_0^S and F_0^T , whose 'mixed' relation with G_0^S and G_0^T can be easily read off from eq.(52) itself (the equality holding term by term).

In the present case, we modify the definition (28) of the θ angle (and the phases) in the following way (consistently with eq.(52)):

$$\begin{aligned} \frac{1}{Y} \left(F_0^S - \frac{\delta_{GS}}{8\pi^2 (T + T^*)} F_0^T \right) &= \sqrt{3} C m_{3/2} e^{i\alpha_S} \sin\theta , \\ \frac{\sqrt{3}}{T + T^*} \left(1 - \frac{\delta_{GS}}{24\pi^2 Y} \right)^{1/2} F_0^T &= \sqrt{3} C m_{3/2} e^{i\alpha_T} \cos\theta . \end{aligned} \quad (53)$$

Notice that eqs.(52) and (53) reduce to eqs.(27) and (28) in the limit $\delta_{GS} = 0$, as they should.

After computing again the soft terms, one finds that the resulting expressions can in practice be obtained²⁶ from the previous formulae (44,45,46,48,49) by making the replacements (50) and

$$\cos\theta \longrightarrow \left(1 - \frac{\delta_{GS}}{24\pi^2 Y} \right)^{-1/2} \cos\theta \quad (54)$$

(without changing $\sin\theta$). The same happens with the formulae for the B_μ parameter eq.(47) but also including⁵⁰ a term $-\frac{\delta_{GS}}{8\pi^2}\frac{\mu^S}{\mu}$ inside the parenthesis that multiplies to $\cos\theta$.

We anticipate that the corrections due to S - T mixing are normally negligible for not too large δ_{GS} . However *they turn out to be important in the $\sin\theta \rightarrow 0$ limit* in which all tree level masses become small and one-loop effects cannot be neglected.

Notice that now F_S is non-vanishing when $\sin\theta \rightarrow 0$, differently from the case without $S - T$ mixing (see eq.(28)). However, if δ_{GS} is not too large, this limit still corresponds to a modulus dominated SUSY-breaking. Indeed, from eq.(53) one obtains

$$\left|\frac{F_0^S}{F_0^T}\right| = \frac{|\delta_{GS}|}{8\pi^2(T+T^*)} \ll 1. \quad (55)$$

The case with several moduli

In the case with several moduli (T_m) the situation is more cumbersome and one is forced to define new goldstino angles. This was first done in section 8 of ref.²⁶ in a different context (extra matter fields). Following this line, the VEV of the scalar potential (see eq(27)) gets modified as

$$V_0 = G_{0S}^S |F_0^S|^2 + \sum_m G_{0T_m}^{T_m} |F_0^{T_m}|^2 - 3e^{G_0} \quad (56)$$

and eq.(28) is modified to^{64,65}

$$\begin{aligned} (G_{0S}^S)^{1/2} F_0^S &= \sqrt{3}Cm_{3/2} e^{i\alpha_S} \sin\theta, \\ (G_{0T_m}^{T_m})^{1/2} F_0^{T_m} &= \sqrt{3}Cm_{3/2} e^{i\alpha_{T_m}} \cos\theta \Delta_m. \end{aligned} \quad (57)$$

where $\sum_m \Delta_m^2 = 1$. For instead, for $m = 1, \dots, 4$ (this is e.g. the case of some Z_N and $Z_N \times Z_M$ orbifolds with three diagonal (1,1) moduli (T_1, T_2, T_3) and one (2,1) moduli ($T_4 = U$)), three new goldstino angles are necessary: $\Delta_1 = \cos\theta_1$, $\Delta_2 = \sin\theta_1 \cos\theta_2$, $\Delta_3 = \sin\theta_1 \sin\theta_2 \cos\theta_3$, $\Delta_4 = \sin\theta_1 \sin\theta_2 \sin\theta_3$. Now, with these definitions, and taking into account that eq.(38) is modified to

$$K = -\log(S + S^*) - \sum_m \log(T_m + T_m^*) + \sum_i \phi_i \phi_i^* \prod_m (T_m + T_m^*)^{n_i^m}, \quad (58)$$

the computation of the soft terms gives

$$\begin{aligned} M_a &= \sqrt{3}Cm_{3/2} \left(\frac{k_a \text{Re}S}{\text{Re}f_a} e^{-i\alpha_S} \sin\theta \right. \\ &\quad \left. + \cos\theta \sum_m e^{-i\alpha_{T_m}} \frac{(b_a^m - k_a \delta_{GS}^m)(T_m + T_m^*) \hat{G}_2(T_m, T_m^*)}{32\pi^3 \text{Re}f_a} \Delta_m \right), \\ m_i^2 &= m_{3/2}^2 C^2 \left(1 + 3\cos^2\theta \sum_m n_i^m \Delta_m^2 \right) + 2m_{3/2}^2 (C^2 - 1), \end{aligned}$$

$$\begin{aligned}
A_{ijk} &= -\sqrt{3}Cm_{3/2} \left[e^{-i\alpha_S} \sin\theta + \cos\theta \sum_m e^{-i\alpha_{T_m}} \Delta_m \omega_{ijk}(T_m) \right] , \\
\omega_{ijk}(T_m) &= 1 + n_i^m + n_j^m + n_k^m - (T_m + T_m^*) \frac{Y_{ijk}^{T_m}}{Y_{ijk}} ,
\end{aligned} \tag{59}$$

where n_i^m are the modular weights of the matter fields and $b_a^m = -C(G_a) + \sum_i T_a(\phi_i)(1 + 2n_i^m)$. The overall modulus case ($T_1 = T_2 = T_3 = T, T_4 = 0$) soft terms eqs.(44,45,46) are recovered for $\Delta_m = 1/\sqrt{3}, \delta_{GS}^m = \delta_{GS}/3, Y_{ijk}^{T_m} = Y_{ijk}^T/3, m=1,2,3$ and $\Delta_4 = 0$. A more complete analysis including the B-term, one-loop (Superstring) corrections, phenomenological consequences, and a comparison with the overall modulus (T) case can be found in ref.⁵⁰.

3.3.2. Large- T limit of Calabi–Yau compactifications

Little is known about the general form of the Kähler potential and couplings of generic Calabi–Yau (2, 2) compactifications. Only a few examples (most notably, the quintic in CP^4) have been worked out⁶⁶ in some detail⁶⁷ and show formidable complexity due to the world-sheet instanton contributions to the Kähler potential. On the other hand, a few generic facts concerning these models are known for the large- T limit^{67,68}. Large T , in practice, does not really mean $T \rightarrow \infty$, since the world-sheet instanton corrections are exponentially suppressed. For values $|T| \geq 2-3$ these world-sheet instanton contributions can often be neglected and, in this sense these T -values are already large. It is true that $|T|$ cannot be infinitely large, since otherwise the quantum corrections to the gauge coupling constants (string threshold corrections) may be too large and spoil perturbation theory. The maximum allowed $|T|$ not spoiling perturbation theory is something which is model dependent but is expected to be much bigger than one since, after all, the threshold corrections are loop effects. In explicit orbifold examples it was found in refs.^{69,32} that $|T| \leq 20-30$ is enough to remain in the perturbative regime. When we talk about the large T limit in what follows we will thus assume T -values which do not spoil perturbativeness. In this limit the Kähler potential K gets a particularly simple form:

$$K(T \rightarrow \infty) = -\log(S + S^*) - 3\log(T + T^*) + \sum_i (T + T^*)^{-1} \phi_i \phi_i^* . \tag{60}$$

Notice that, the resulting Kähler potential is analogous to the one obtained in orbifold models eq.(38) with matter fields in the untwisted sector. Therefore we can use eqs.(44,45,46,47,48,49) with $n_i = -1$ in order to obtain $T \rightarrow \infty$ soft terms:

$$M_a = \sqrt{3}Cm_{3/2} \frac{k_a \text{Re}S}{\text{Re}f_a} e^{-i\alpha_S} \sin\theta , \tag{61}$$

$$m_i^2 = m_{3/2}^2 C^2 \sin^2\theta + 2m_{3/2}^2 (C^2 - 1) , \tag{62}$$

$$A_{ijk} = -\sqrt{3}Cm_{3/2} [e^{-i\alpha_S} \sin\theta + e^{-i\alpha_T} \cos\theta \omega_{ijk}(T, T^*)] ,$$

$$\omega_{ijk}(T, T^*) = -\frac{(T + T^*)}{\sqrt{3}} \frac{Y_{ijk}^T}{Y_{ijk}}, \quad (63)$$

$$B_\mu = m_{3/2} \left[-1 - C\sqrt{3}e^{-i\alpha_S} \sin\theta \left(1 - \frac{\mu^S}{\mu}(S + S^*) \right) - Ce^{-i\alpha_T} \cos\theta \left(1 - \frac{\mu^T}{\mu}(T + T^*) \right) \right], \quad (64)$$

$$B_\lambda = m_{3/2} \left[(3C^2 - 1) + Ce^{-i\alpha_T} \cos\theta \left(2 + \frac{\lambda^T}{\lambda}(T + T^*) \right) \right], \quad (65)$$

$$B_Z = \frac{m_{3/2}}{X} [(3C^2 - 1) + 2C \cos\alpha_T \cos\theta], \quad (66)$$

$$X \equiv 1 + Ce^{i\alpha_T} \cos\theta,$$

where we have ignored the possible one-loop corrections to these formulae.

It can be further argued that, in the large T -limit, the non-vanishing Yukawa couplings tend (exponentially) to constants, as computed in specific examples. Then one can take $\omega_{ijk} \rightarrow 0$ in the mentioned limit. Of course, this simplifies the phenomenological analysis.

It is interesting to remark that in this large- T limit of Calabi–Yau-type compactifications the results obtained for the soft scalar and gaugino masses and A-parameters are quite similar to those in eq.(37) obtained in a model-independent manner for $\sin\theta = 1$. The role of dimensionful parameter is played now by $m_{3/2} \sin\theta$ (for $C = 1$) instead of simply $m_{3/2}$. Thus *dilaton-dominated SUSY-breaking is not the only situation in which universal soft scalar masses are obtained*, as the present model exemplifies. Anyhow we point out that in the case with several moduli the situation might be much more cumbersome and one is forced to define new goldstino angles (as we did in the orbifold case in the previous subsection). If we allow generic values for these angles we obtain a deviation from the previous universal behaviour (see e.g. the soft scalar masses in eq.(59) for the orbifold case).

It is also interesting to notice how, for $C = 1$, all these terms tend to zero at the same speed as $\sin\theta \rightarrow 0$, even for a finite value of $m_{3/2}$. Indeed, for a very small $\sin\theta$ the gravitino mass $m_{3/2}$ decouples from the SUSY-breaking soft terms and may become much larger than them. However, for $\sin\theta = 0$ the one-loop corrections to the Kähler potential cannot probably be neglected (unfortunately, the one-loop corrections to the Kähler potential in the large- T limit of Calabi–Yau compactifications are unknown), and care should be taken before getting any definite conclusion (see the case of one-loop orbifold corrections discussed above).

The above statements concerning the large T -limit of Calabi–Yau compactifications are known to be true for $(2, 2)$ models, which yield a gauge group $E_6 \times E_8$. In order to make contact with the standard model one has to break this structure with Wilson line gauge symmetry breaking and/or use $(0, 2)$ type compactifications. However, it is reasonable to expect that the general structure in eq.(60) will still apply in these more complicated cases and, hence, eqs.(61,62,63,64,65,66) will still hold.

Notice that the $\sin\theta \rightarrow 0$ limit of the large- T Calabi–Yau Superstrings is different

from the "no-scale" supergravity models discussed in the literature⁷⁰. Although in both models (for $C=1$) one has at the tree level $m_i = A = 0$, the behaviour of the gaugino mass is totally different. In the no-scale models the gaugino mass is non-vanishing and constitutes the only source of SUSY-breaking whereas in the present class of models the gaugino mass also vanishes.

3.3.3. Supersymmetric-spectra expected in Superstring models

The formulae for soft terms written in the previous subsections may lead to different phenomenological situations²⁶ depending on, e.g., the phases, the value of the cosmological constant, the possible values of the modular weights of the particles, the ansatz for the B -parameter, etc. In what follows we will assume vanishing phases and cosmological constant.^{‡‡} The former is consistent with the experimental limits on the EDMN and the latter with the experimental constraints in present cosmology. In order to have $V_0 = 0$ one may assume that there is *some yet undiscovered dynamics which guarantees that $V_0 = 0$ at the minimum*. In our context this implies that the dynamics of the S and T superfields is such that their auxiliary fields break supersymmetry with vanishing cosmological constant. This could come about e.g. if some non-perturbative dynamics generates an appropriate superpotential $W(S, T)$ with this $V_0 = 0$ property. Indeed, such type of superpotentials can be constructed (see ref.^{31,34}), although the physical origin of them is certainly obscure. Anyway, if one adopts this philosophy one must set $V_0 = 0$ (or, equivalently, $C = 1$) in all the expressions for the soft terms.

First, let us compare the soft gaugino and scalar masses in the overall modulus case. From eqs.(44,45) these are given roughly by

$$M_a^2 = 3 m_{3/2}^2 \sin^2 \theta, \quad (67)$$

$$m_i^2 = m_{3/2}^2 (1 + n_i \cos^2 \theta); \quad n_i = -1, -2, -3, \dots, \quad (68)$$

where we have neglected small threshold effect corrections ($ReS \simeq Ref_a$). With respect to the one-loop correction (50), which only appears in gaugino masses, it turns out to be also irrelevant due to the fact that δ_{GS} is usually small. In all orbifold models considered up to now δ_{GS} is of the same order of magnitude as the b' -coefficients appearing in the model. Both the one-loop (S - T mixing) corrections eq.(54) and the one-loop gaugino term proportional to $\cos \theta$ in eq.(44) are also neglected. As was already mentioned in subsection 3.3.1, the latter turn out to be numerically irrelevant for not too small $\sin \theta$. This is usually the situation since

$$\cos^2 \theta \leq \frac{1}{|n_i|} \quad (69)$$

in order to avoid negative mass-squared of the scalars of modular weight n_i (see eq.(68)). E.g. $n_i = -3$ implies $\sin \theta \geq 0.82$. Thus the allowed values of $\sin \theta$ are

^{‡‡}The impact of a non-vanishing tree-level cosmological constant V_0 on SUSY-spectra has been studied in ref.⁵⁵.

relatively close to one and *the dilaton is necessarily the dominant source of SUSY-breaking*. Notice, however, that the moduli contribution to SUSY-breaking is *not* in general negligible. Of course, for $|\sin \theta| = 1$ one recovers the dilaton-dominated SUSY-breaking results. Now, eqs.(67,68) clearly imply that soft gaugino masses (using the relation $\sin^2 \theta = 1 - \cos^2 \theta$) are bigger than soft scalar masses

$$M_a > m_i . \quad (70)$$

Let us now discuss the predictions for the low-energy ($\sim M_Z$) sparticle spectra. There are several particles whose mass is rather independent of the details of $SU(2)_L \times U(1)_Y$ breaking and is mostly given by the boundary conditions and the renormalization group running. In particular, in the approximation that we will use (neglecting all Yukawa couplings except the one of the top), that is the case of the gluino g , all the squarks (except stops and left sbottom) $Q_L = (u_L, d_L), u_L^c, d_L^c$ and all the sleptons $L_L = (v_L, e_L), e_L^c$. For all these particles one can write explicit expressions for the masses in terms of the gravitino mass and $\sin \theta$.

$$M_g^2(M_Z) = 9.8 M^2 = 29.4 m_{3/2}^2 \sin^2 \theta , \quad (71)$$

$$m_{Q_L}^2(M_Z) = m_{Q_L}^2 + 8.3 M^2 = m_{3/2}^2 (1 + n_{Q_L} \cos^2 \theta + 25 \sin^2 \theta) , \quad (72)$$

$$m_{u_L^c, d_L^c}^2(M_Z) = m_{u_L^c, d_L^c}^2 + 8 M^2 = m_{3/2}^2 (1 + n_{u_L^c, d_L^c} \cos^2 \theta + 24 \sin^2 \theta) , \quad (73)$$

$$m_{L_L}^2(M_Z) = m_{L_L}^2 + 0.7 M^2 = m_{3/2}^2 (1 + n_{L_L} \cos^2 \theta + 2 \sin^2 \theta) , \quad (74)$$

$$m_{e_L^c}^2(M_Z) = m_{e_L^c}^2 + 0.23 M^2 = m_{3/2}^2 (1 + n_{e_L^c} \cos^2 \theta + 0.7 \sin^2 \theta) , \quad (75)$$

where the last term in eqs.(72,73,74,75) gives the effect of gaugino loop contributions in the low-energy running. In the previous formulae we have neglected the scalar potential D-term contributions which are normally small and the contribution to the scalar mass RGEs of the $U(1)_Y$ D-term. These may be found in eq.(9) of ref.⁷¹. Now, the low-energy mass relations turn out to be

$$m_l < m_q \simeq M_g . \quad (76)$$

since the low-energy scalar masses are mainly determined by the gaugino contributions. (This also implies that even with non-vanishing V_0 , these results are still maintained.) The slepton masses are smaller than squark masses because they do not feel the important gluino contribution.

Let us consider two examples in order to analyze the previous relations in more detail. In the first example we will assume different (flavour-independent) modular weights for the different squark and sleptons within each generation. The modular weights are chosen so that one can have appropriate large string threshold corrections to fit the joining of gauge coupling constants at a scale $\simeq 10^{16}$ GeV (see refs.^{69,32}).

$$n_{Q_L} = n_{d_L^c} = -1 , \quad n_{u_L^c} = -2 , \quad n_{L_L} = n_{e_L^c} = -3 , \quad n_{H_1} + n_{H_2} = -5, -4 . \quad (77)$$

The above values together with a $ReT \simeq 16$ lead to good agreement for $\sin^2 \theta_W$ and α_3 . This scenario is also interesting because it provides us with an explicit model with

non-universal scalar masses and shows the general features of models with some of the modular weights different from -1 . For the sake of definiteness, in the following we will focus on the case $n_{H_1} = -2, n_{H_2} = -3$. Other possible choices do not lead to significant modifications in the phenomenological results. For the masses of scalar particles with modular weights $-1, -2$ and -3 , eq.(68) gives respectively

$$m_{-1}^2 = m_{3/2}^2 \sin^2 \theta, \quad m_{-2}^2 = m_{3/2}^2 (1 - 2 \cos^2 \theta), \quad m_{-3}^2 = m_{3/2}^2 (1 - 3 \cos^2 \theta). \quad (78)$$

Several comments are in order. First of all the goldstino angle is constrained to $\sin^2 \theta \geq 2/3$, otherwise the scalars with modular weight -3 (the sleptons and the Higgs field H_2 in the present case) would get negative squared masses. For $|\sin \theta| = 1$ one recovers the dilaton dominated SUSY-breaking results. However, as one gets away from that value important deviations occur. The scalars with large (negative) modular weights get soft masses substantially smaller than those with e.g. $n_i = -1$. In particular, the sleptons are much lighter than the squarks already at the string scale for $|\sin \theta| \simeq 0.8$. Therefore we have to give the masses of the gluino, the squarks and the sleptons for a fixed value of θ . In particular, we have chosen the smallest possible value $\sin^2 \theta = 2/3$. For such value of θ the contribution of the moduli field T to SUSY-breaking is substantial. The mass ratios now turn out to be

$$M_g : m_{Q_L} : m_{u_L^c} : m_{d_L^c} : m_{L_L} : m_{e_L^c} \simeq 1 : 0.95 : 0.92 : 0.92 : 0.25 : 0.11, \quad (79)$$

where we have included the small contribution to the scalar mass RGEs of the $U(1)_Y$ D-term. The results are qualitatively similar to those of the second scenario (see below), in spite of the different set of soft scalar masses, because the low-energy scalar masses are mainly determined by the gaugino contributions. The only exception is the e_L^c mass, which only feels the small B -ino contribution. The similarity with eq.(80) is clear (the results would be still more similar for the other possible values of θ). Again, the masses do not contain the scalar potential D-term contributions, which depend on the process of symmetry breaking and anyway are very small. In particular, concerning the possible shifts of the slepton masses, similar remarks as in the second example apply (see below).

In the second scenario we will assume that all particles have modular weight -1 , as in the large- T Calabi-Yau limit and possible orbifold Superstring scenarios. M_a and m_i scale, to a first approximation like $m_{3/2} \sin \theta$ (see eqs.(67,68)) thus the gluino and the squark and slepton masses are in the universal ratio (at the M_Z scale)

$$M_g : m_{Q_L} : m_{u_L^c} : m_{d_L^c} : m_{L_L} : m_{e_L^c} \simeq 1 : 0.94 : 0.92 : 0.92 : 0.32 : 0.24. \quad (80)$$

Although squarks and sleptons have the same soft mass, at low-energy the former are much heavier than the latter because of the gluino contribution to the renormalization of their masses (see eqs.(72,73,74,75)). Actually, the result corresponds to scalar masses without D-term contributions. The latter depend on $\tan \beta \equiv \frac{\langle H_2 \rangle}{\langle H_1 \rangle}$, which in turn depends on the details of $SU(2)_L \times U(1)_Y$ breaking. However, as mentioned above, the inclusion of D-terms leads only to small shifts. In particular, for the

extreme case of maximum D-term contributions (large $\tan\beta$), the only modification is that the e_L and e_L^c masses get slightly shifted upwards (a few GeV) whereas the v_L mass gets lowered (even below e_L^c , if the gluino is lighter than 350 GeV). The contribution to the scalar mass RGEs of the $U(1)_Y$ D-term are vanishing due to their universal boundary conditions.

The rest of the supersymmetric mass spectra are more dependent on the $SU(2)_L \times U(1)_Y$ breaking process and the value assumed for the B parameter. (The A -parameter which appears through the RGEs is given by eq.(63) with $\omega_{ijk} = 0$ in the large- T limit of Calabi–Yau compactifications as explained in subsection 3.3.2. In the orbifold case is given by eq.(46) with the last term in ω_{ijk} vanishing as explained in subsection 3.3.1.) Indeed, these scenarios are five-parameter models in general: $m_{3/2}$, $\sin\theta$ (recall that one is restricted to a region fulfilling eq.(69)), B , the top-quark mass m_t and the $\hat{\mu}$ parameter. One can eliminate one of them (e.g. $\hat{\mu}$, which is the one we know the least) in terms of the others by imposing appropriate symmetry breaking at the weak scale. The value of the top mass is quite constrained by the LEP and CDF data so that m_t is not a source of big uncertainty. However, B introduces some source of uncertainty. In general, B is a model dependent function of $m_{3/2}, \sin\theta, T$, etc., as discussed in subsection 3.1. One could leave B as a free parameter in the analysis, but we find more interesting to display the results in terms of some reasonable ansatz for the $\sin\theta$ dependence of the B parameter. One possibility is to use B_μ (eq.(47)) with the simplifying assumption $\mu^S/\mu = \mu^T/\mu = 0$. This analysis can be found in section 7 of ref²⁶. But we think it is still more interesting to use any of the two mechanisms to generate the μ term discussed in section 2 and 3.1. Now μ is no longer a free parameter and $B_{\lambda,Z}$ are completely determined (see eqs.(48,49). The analysis using B_Z can be found in ref.⁵⁰.

Thus *we have traded* the four free soft parameters (M, m, A, B) of the MSSM by the two parameters $m_{3/2}$ and θ .

The $\sin\theta \rightarrow 0$ (modulus-dominated) limit

There is only one situation in which *the gaugino masses may become smaller than the scalar masses*: a very small $\sin\theta$. We recall that this limit corresponds to a modulus-dominated SUSY-breaking even if S - T mixing is present, as discussed above eq.(55). This limit *is only accesible if* all modular weights of sparticles are equal to -1 (see eq.(69)), as in the large- T Calabi–Yau limit and possible orbifold Superstring scenarios. In this case, $\sin\theta \rightarrow 0$ implies $M_a, m_i \rightarrow 0$ as can be obtained from eqs.(67,68) and therefore, as $\sin\theta$ decreases, one can ignore less and less the one-loop corrections to the Kähler potential. Furthermore, the results get more and more dependent on the form of the Superstring threshold correction function $f_a(T)$ (see eq.(26)), which is still quite uncertain in the context of Calabi–Yau-type compactifications. On the other hand, as we studied already in subsection 3.3.1, both the one-loop threshold effects and the one-loop corrections to the Kähler potential are much better known in the context of orbifold four-dimensional Superstrings. Thus it makes sense to study the orbifold analogous to the Calabi–Yau scenario, which will explicitly provide us with one-loop corrected expressions for the soft terms.

Let us now describe the form of the soft masses in this scenario, starting with the scalar masses. They can be obtained from eq.(68) with $n_i = -1$ after introducing the one-loop correction eq.(54). One gets

$$m_i^2 = m_{3/2}^2 \left[1 - \left(1 - \frac{\delta_{GS}}{24\pi^2 Y} \right)^{-1} \cos^2 \theta \right], \quad (81)$$

where Y was defined in eq.(50). This result is numerically very similar to that studied above with $n_i = -1$ as long as $\sin \theta$ is not much smaller than one. However, in the $\sin \theta \rightarrow 0$ limit in which the tree-level scalar masses vanish one finds

$$m_i^2 (\sin \theta \rightarrow 0) \simeq m_{3/2}^2 \left(\frac{-\delta_{GS}}{24\pi^2 Y} \right) \simeq m_{3/2}^2 (-\delta_{GS}) 10^{-3}. \quad (82)$$

We thus observe that, in the case of orbifolds, the inclusion of the one-loop corrections in the Kähler potential has the effect of "regulating" in some way the $\sin \theta \rightarrow 0$ limit yielding a non-vanishing result for the scalar masses.

Finally, concerning the gaugino masses, they can be obtained from eq.(44), with $C = 1$ and vanishing phases, after introducing the one-loop correction from eqs.(50,54). One gets

$$M_a = \frac{k_a Y}{2 \text{Re} f_a} \sqrt{3} m_{3/2} [\sin \theta + \frac{(b'_a - k_a \delta_{GS})(T + T^*) \hat{G}_2(T, T^*)}{16 \sqrt{3} \pi^3 k_a Y} \left(1 - \frac{\delta_{GS}}{24\pi^2 Y} \right)^{-1/2} \cos \theta]. \quad (83)$$

Since all modular weights are equal to -1 , one has $b'_a = b_a$ (see eq.(43)). Then, using a value $\text{Re} T \simeq 1.2$ (this value is suggested by several gaugino condensation analyses, see e.g. ref.²⁷ and references therein) close to the duality self-dual point, which is what one would normally expect in a duality invariant theory, one obtains the following numerical results:

$$\begin{aligned} M_3 &\simeq 1.0 \sqrt{3} m_{3/2} [\sin \theta - (3 + \delta_{GS}) 4.6 \times 10^{-4} \cos \theta], \\ M_2 &\simeq 1.06 \sqrt{3} m_{3/2} [\sin \theta - (-1 + \delta_{GS}) 4.6 \times 10^{-4} \cos \theta], \\ M_1 &\simeq 1.18 \sqrt{3} m_{3/2} \left[\sin \theta - \left(\frac{-33}{5} + \delta_{GS} \right) 4.6 \times 10^{-4} \cos \theta \right]. \end{aligned} \quad (84)$$

We are really interested in understanding the qualitative behaviour of this small $\sin \theta$ limit and hence we will just take a fixed value for δ_{GS} , e.g. $\delta_{GS} = -5$. This is a negative integer with a magnitude of order of the b' coefficients involved and hence it is not an unreasonable value. We will comment below what happens as we vary this parameter. The most prominent feature of the soft terms is that *for values of $|\sin \theta|$ below 5×10^{-2} the gaugino masses become smaller than the scalar masses.*

$$M_a < m_i. \quad (85)$$

This is something which is qualitatively different from the previous results, where gaugino masses were always necessarily larger than scalar masses. On the other side, this situation is quite similar to the one obtained in explicit gaugino condensation models²⁸ although it is not really identical (for a comparison of both situations, see section 8 of ref.²⁶). Indeed, in this scenario as $\sin \theta$ decreases the gaugino/squark mass ratio decreases.

Notice that the qualitative behaviour found here for small $\sin \theta$ is generic for any non-vanishing negative integer δ_{GS} . The only difference is the particular value of $\sin \theta$ at which the gaugino masses start being smaller than the scalar masses. Also, different values for δ_{GS} lead to different gaugino mass ratios (e.g. M_3/M_2) as $\sin \theta \rightarrow 0$, but we consider this as a small correction to the most relevant feature found in this limit, which is that the gaugino masses become small compared to the scalar masses. (The case $\delta_{GS} = 0$ is special since, as can be seen from eq.(82), the scalar masses tend to zero and the gaugino masses provide essentially the only source of SUSY-breaking for $\sin \theta = 0$. $\delta_{GS} = 0$ is e.g. the case of the orbifold $Z_2 \times Z_2$, however, generically there will be S - T mixing in the Kähler potential and a case with $\delta_{GS} = 0$ is atypical.) Another point to remark is that in the present limit the gravitino mass is much larger (more than an order of magnitude bigger) than the soft masses. For example, for $\delta_{GS} = -5$, eq.(82) implies that $m_{3/2} \simeq 14m_i$.

Let us describe now what is the structure of the low-energy SUSY-spectra in this small $\sin \theta$ limit. Let us start as usual with the sector of the spectrum which is rather insensitive to the radiative electroweak breaking, i.e. the gluino, the squarks (except stops and left sbottom) and the sleptons. Since the soft terms have a different dependence on θ , we will content ourselves with showing results for a fixed small value of $\sin \theta$, because this is the limit we want to explore (for large $\sin \theta$ the results correspond to a good approximation with those studied above for $n_i = -1$). For the illustrative choice $\theta - \pi = 5 \times 10^{-3}$, with $ReT \simeq 1.2$ and $\delta_{GS} = -5$, the situation now is completely reversed with respect to the above one. The gluino is substantially lighter than the scalars.

$$M_g < m_l \simeq m_q. \quad (86)$$

For example, in this particular case the relation is $m_{q,l} \simeq 2.5M_g$. Notice also that the physical masses of squarks and sleptons are almost degenerate. This happens because the universality of soft scalar masses at high energy is not destroyed by the gluino contribution to the mass renormalization, which is now very small.

The above results tell us that *if gluinos lighter than squarks and sleptons are found, this could be an indication that the dominant source of SUSY-breaking lies in the moduli* and not in the dilaton sector. Concerning the rest of the spectrum, similar remarks as above apply.

Discussion of the overall Supersymmetric-spectra

Let us try to summarize the most prominent patterns obtained for the spectra of supersymmetric particles in this large class of models. For a given choice of Superstring model the free soft parameters of the MSSM (M, m, A, B) are *given in terms of the*

gravitino mass $m_{3/2}$ and the goldstino angle θ . In some Superstring models in which the one-loop corrections become important additional dependence on other parameters (δ_{GS} , ReT) may appear, although the latter are less crucial in understanding the qualitative patterns of soft terms.

One first point to remark is that one can have flavour-independent soft scalar masses even without dilaton-dominated SUSY-breaking. In fact, all the scenarios discussed above have flavour-independent scalar masses, although in some case different scalars within the same flavour generation can have different masses. Thus *dilaton-dominated SUSY-breaking is a sufficient but not necessary condition to obtain scalar mass universality.*

For goldstino angle $|\sin \theta| \geq 5 \times 10^{-2}$ the results for the different scenarios are not terribly different. The heaviest particles are the coloured ones with gluinos and squarks almost degenerate and 3 to 6 times heavier than sleptons.

For very small $\sin \theta$ one can no longer neglect in general the one-loop corrections. Now the situation concerning the spectrum is very much changed and the gluino may be even lighter than the squarks and sleptons. The latter become almost degenerate. If a spectrum of this type is found, it could be an indication of a modulus-dominated SUSY-breaking.

It is also important to recall that values $\sin^2 \theta \leq 1/2$ are only possible in models in which all modular weights are equal to -1 . Otherwise some of the squarks and/or sleptons would get negative squared masses at the Superstring scale.

The case with several moduli

The formulae for soft terms in the case of several moduli were written at the end of subsection 3.3.1 (see eq.(59)). Taking into account these results is possible to see that the general phenomenological conclusions obtained above for the overall modulus case may be somewhat modified⁵⁰. For example, particles with $n_i = -1$ may also get negative mass-squared for some choices of the angles. Also, scalar masses may become bigger than gaugino masses even at tree level. For an extended discussion see ref.⁵⁰.

Final comment about the spectra

The general pattern of SUSY-spectra found in the present approach are very characteristic. Optimistically, if the spectrum of SUSY particles is eventually found, one will be able to rule out (or rule in) some of the general scenarios (e.g., dilaton or modulus dominance) here discussed. More modestly, we hope that the formulae and the examples worked out in this paper will be of help in looking for a more fundamental understanding of the origin of SUSY-breaking soft terms in the Supersymmetric Standard Model.

4. Acknowledgements

We thank A. Lleyda for her valuable help.

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